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## GEOMETRY.

85. Proposed by S. F. NORRIS, Professor of Astronomy and Mathematics, Baltimore City College, Baltimore, Md.

Prove by pure geometry. Give direct proof, if possible.

If the bisectors of two angles of a triangle are equal, the triangle is isosceles.

[From *Wentworth's Plane Geometry*, exercise 43, page 72.]

I. Solution by J. M. COLAW, A. M., Monterey, Va., and EDMUND FISH, Hillsboro, Ill.

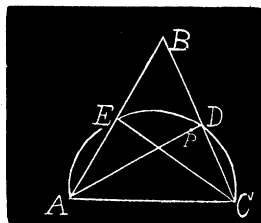
In triangle  $ABC$ , let  $AD$ ,  $CE$ , be the bisectors of angles  $A$  and  $C$ , and  $AD = CE$ . Then angle  $A = \text{angle } C$ , and triangle  $ABC$  is isosceles.

Suppose a circle passed through  $A$ ,  $C$ , and  $E$ . It will also pass through  $D$ . If not, suppose that it cut  $AD$  in any point  $P$  short of  $D$ . Then arc  $EB > \text{arc } PE$ , since  $\angle ECA (= \angle DCE) > \angle PCE$ .

Also, arc  $PE = \text{arc } PC$ , since  $\angle EAP = \angle PAC$ . Whence arc  $AEP > \text{arc } EPC$ .

$\therefore$  chord  $BP > \text{chord } CE$ . But by hypothesis,  $AD = CE$ .  $\therefore AP > AD$ , which is absurd. In the same way it may be shown that the supposition that the circle, which passes through  $A$ ,  $C$ , and  $E$ , cuts  $BD$  in any point  $P'$ , beyond  $D$ , also leads to an absurdity. The circle must therefore pass through  $D$ . Hence  $\angle EAD (= \frac{1}{2} \angle A) = \angle DCE (= \frac{1}{2} \angle C)$ .

$\therefore \angle A = \angle C$ , and triangle  $ABC$  is isosceles.



II. Solution by OTTO CLAYTON, Teacher of Mathematics and Physics, Remington High School, Remington, Ind.

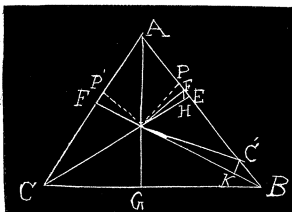
Draw the bisector  $AG$  meeting the given bisectors  $CE$  and  $BF$  of the given triangle  $ABC$ , in the point  $O$ . Revolve  $ACG$  about  $AG$  as an axis until  $AC$  coincides with  $AB$ . Then the point  $C$  will fall within the segment  $AB$ , on the point  $B$ , or without the segment  $AB$ ; according as  $\angle ACO$  is greater, equal to, or less than  $\angle ABO$ . And the point  $F$  will fall within  $AE$ , on the point  $E$  or within  $EB$ , according as  $\angle ACO$  is greater, equal to, or less than  $\angle ABO$ .

But  $\angle ACO$  cannot be greater than  $\angle ABO$ , for  $(C'O + OE) = CE$  would be less than  $(BO + OF') = BF$ , which is contrary to the hypothesis that  $CE = BF$ .

Likewise  $\angle ACO$  can not be less than  $\angle ABO$ , for  $(B'O + OE) = CE$  would be greater than  $(BO + OF) = BF$ , which is contrary to the hypothesis. Therefore  $\angle ACO = \angle ABO$ , and  $C$  falls upon  $B$ . Therefore the triangle is isosceles.

I think this is a simple proof and does not involve anything outside of the first book of Wentworth.

In the proof I did not show how  $OE + OC'$  is less than  $BO + OF'$ . When the perpendicular  $OP$  falls between  $OE$  and  $OB$  the reason is obvious. When  $OP$  falls without, construct equilateral triangles  $F'OH$  and  $C'OK$ . Then prove  $HE$  less than  $KB$ .



### III. Solution by the PROPOSER.

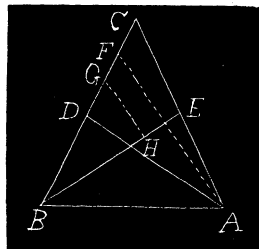
Let the bisectors  $BE$  and  $AD$  be equal, prove triangle  $ABC$  isosceles.

Three suppositions are possible.

1st,  $\angle A > \angle B$ ; 2nd,  $\angle A < \angle B$ ; 3rd,  $\angle A = \angle B$ .

First, suppose  $\angle A > \angle B$ : then  $\frac{1}{2}\angle A > \frac{1}{2}\angle B$ .

Construct  $\angle FAD = \angle CBE$ . Then in the triangle  $FAB$ ,  $FB > FA$  (greater side opposite greater angle). Lay off on  $BF$  a distance  $BG$  equal to  $AF$ , and draw  $GH$  parallel to  $FA$ . Then the triangle  $BGH =$  triangle  $FAD$  ( $BG = FA$ , by construction,  $\angle GBH = \angle FAD$ , for the same reason,  $\angle BGH = \angle DFA$ , exterior-interior angles.)



$\therefore DH = BA$  (homologous sides of equal triangles) *which is absurd*, because  $BE = AD$ , by hypothesis, and  $BH$  is only a part of  $BE$ .

Second, in a similar manner it can be shown that  $\angle B$  cannot be greater than  $A$ ; i. e.  $\angle A$  cannot be less than  $\angle B$ .

Third, as  $\angle A$  can neither be greater nor less than  $\angle B$ , it must be equal to  $\angle B$ .  $\therefore$  the triangle is isosceles. Q. E. D.

For other demonstrations of this problem, see Vol. II., pages 158, 189—192. EDITOR.

86. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Prove that the four conics which have  $S$  for focus and which touch the three sides of each of the triangles  $ABC$ ,  $AEF$ ,  $BFD$ ,  $CDE$ , have their latera-recta equal.

Solution by the PROPOSER.

Reciprocate with respect to  $S$ ; then we have the three altitudes of an equilateral triangle passing through a point, and the circumscribing circles of the four triangles formed by joining the feet of the perpendiculars equal.

The latera recta of the given conics are then equal.

87. Proposed by WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military Academy, Washington, Miss.

Given any two straight lines in space,  $AB$ ,  $CD$ , which do not intersect. So construct upon one of the lines as base, a triangle, having its vertex in the other line, such that its perimeter shall be a minimum.

No solution of this problem has been received.

88. Proposed by FREDERICK R. HONEY, Ph. B., Instructor in Mathematics in Trinity College, New Haven, Conn.

Prove that the volume of the frustum of a cone is equal to one-sixth of the altitude multiplied by the sum of the areas of the upper base, the lower base, and four times the area of the section midway between the upper and lower bases.

Solution by FREMONT CRANE, Sand Coules, Mont.; ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa; G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va., and the PROPOSER.

Let  $R$  = radius of the lower base;  $r$  = radius of the upper base;  $\rho$  = radius